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MULTIPLE SOFT FAULT DIAGNOSIS OF NONLINEAR DC CIRCUITS CONSIDERING COMPONENT TOLERANCES

Michał Tadeusiewicz, Stanisław Hałgas

Technical University of Łódź, ul. Stefanowskiego 18/22, 90-924 Łódź, Poland (✉michal.tadeusiewicz@p.lodz.pl)

Abstract

This paper is devoted to multiple soft fault diagnosis of analog nonlinear circuits. A two-stage algorithm is offered enabling us to locate the faulty circuit components and evaluate their values, considering the component tolerances. At first a preliminary diagnostic procedure is performed, under the assumption that the non-faulty components have nominal values, leading to approximate and tentative results. Then, they are corrected, taking into account the fact that the non-faulty components can assume arbitrary values within their tolerance ranges. This stage of the algorithm is carried out using the linear programming method. As a result some ranges are obtained including possible values of the faulty components. The proposed approach is illustrated with two numerical examples.

Keywords: analog circuits, fault diagnosis, multiple faults, nonlinear circuits.

1. Introduction

Fault diagnosis of analog circuits is an open problem for design validation and testing of electronic devices [1-30]. A fault can be catastrophic (hard) if it leads to some topological changes (open circuit or short circuit) or soft if a parameter drifted from its tolerance range. Fault diagnosis includes detecting faulty circuits, locating faulty components and evaluating their values. If most circuit simulations take place before any testing, the diagnostic method is classified as the simulation-before-test (SBT) approach, otherwise the method is classified as the simulation-after-test (SAT) approach.

Soft fault diagnosis is usually carried out using the SAT approach. Most of the works in this area are limited to a single fault, e.g. [6, 10, 12, 16, 21-23, 27]. Less papers have been devoted to multiple faults, e.g. [8, 11, 13, 17-18, 26]. To perform soft fault diagnosis a diagnostic test is arranged leading to diagnostic equations including circuit parameters as variables. For nonlinear circuits the diagnostic equation usually cannot be formulated in explicit analytical form. In addition, a difficult problem is fault masking due to scattering of the circuit parameters within their tolerance ranges.

This paper deals with multiple soft fault diagnosis of DC nonlinear circuits, takes into account component tolerances and offers a two-stage algorithm. It enables us to locate the faulty components and evaluate their parameters, considering the component tolerances. At first a preliminary diagnostic procedure developed in Section 2 is performed, under the assumption that the non-faulty components have nominal values. The obtained results are approximate and considered as tentative. Then they are corrected, taking into account the fact that the non-faulty components can assume arbitrary values within their tolerance ranges. This stage of the algorithm is performed using the linear programming method described in Section 3. The proposed approach is illustrated with two numerical examples placed in Section 4.

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To perform the first stage of the algorithm a diagnostic test is arranged. For this purpose we apply DC voltage sources to nodes accessible for excitation and measure voltages at accessible nodes. For different values of the voltage sources we obtain n values of the voltages $u_1,\dots,u_n$ and form a vector $\boldsymbol{u}=[u_1 \dots u_n]^{\mathrm{T}}$, where n is the number of circuit components considered as possibly faulty. Let $\boldsymbol{x}=[x_1 \dots x_n]^{\mathrm{T}}$ be a vector of the circuit parameters. Then the test equation has the form

$$\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{u}, \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x})=[f_1(\boldsymbol{x})\dots f_n(\boldsymbol{x})]^{\mathrm{T}}$ is a nonlinear function. Generally, this function is not given in an explicit analytical form. However, the values of $f_i(\boldsymbol{x})$ $(i=1,\dots,n)$ and their derivatives with respect to x_j $(j=1,\dots,n)$ can be found numerically for given values of $x_1,\dots,x_n$.

2. Preliminary multiple fault diagnosis

Let $\boldsymbol{x}^{(0)}$ be a vector composed of nominal values of the circuit components. We expand the function $\boldsymbol{f}(\boldsymbol{x})$ appeared in (1) into the Taylor series about $\boldsymbol{x}^{(0)}$ and neglect the higher order terms, similarly as in [18]

$$\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{f}\left(\boldsymbol{x}^{(0)}\right)+\left[\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}}\left(\boldsymbol{x}^{(0)}\right)\right]\left(\boldsymbol{x}-\boldsymbol{x}^{(0)}\right). \tag{2}$$

Since the function $\boldsymbol{f}(\boldsymbol{x})$ is not given in explicit analytical form we cannot find $\boldsymbol{f}\left(\boldsymbol{x}^{(0)}\right)$ and $\left(\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}}\right)\left(\boldsymbol{x}^{(0)}\right)$ directly. To overcome this drawback we set $\boldsymbol{x}=\boldsymbol{x}^{(0)}$ and carry out the DC and the sensitivity analyses of the circuit. Substituting (1) into (2) yields

$$\boldsymbol{A}\boldsymbol{y}=\boldsymbol{b}, \tag{3}$$

where: $\boldsymbol{A}=\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}}\left(\boldsymbol{x}^{(0)}\right)$ is an $n\times n$ matrix, $\boldsymbol{y}=\boldsymbol{x}-\boldsymbol{x}^{(0)}$ and $\boldsymbol{b}=\boldsymbol{u}-\boldsymbol{f}\left(\boldsymbol{x}^{(0)}\right)$ are n-vectors. The elements of $\boldsymbol{y}$ are deviations of the circuit components from the nominal values. To solve the equation (3) we first determine the rank of $\boldsymbol{A}$ applying the singular value decomposition (SVD) [31]. Let the rank of $\boldsymbol{A}$ be r $(\mathrm{rank}(\boldsymbol{A})=r)$, where $r\le n$. We choose an integer $p\le r$ such that $p=2$ in the case of single fault diagnosis, $p=3$ in the case of double fault diagnosis, and so on. We create all $n\times p$ submatrices of the matrix $\boldsymbol{A}$ having the rank equal to p using the procedure described in the Appendix. Let the number of these submatrices be M and $\boldsymbol{A}^{(i)}$, where $i\in\{1,\dots,M\}$ be an arbitrary matrix belonging to this set. We write the equation

$$\boldsymbol{A}^{(i)}\boldsymbol{y}^{(i)}=\boldsymbol{b}, \tag{4}$$

where $\boldsymbol{y}^{(i)}$ is a vector consisting of p elements corresponding to the columns of the matrix $\boldsymbol{A}$ used to form $\boldsymbol{A}^{(i)}$. To solve the equation (4) we apply the method of normal equation:

$$\left(\boldsymbol{A}^{(i)}\right)^{\mathrm{T}}\boldsymbol{A}^{(i)}\boldsymbol{y}^{(i)}=\left(\boldsymbol{A}^{(i)}\right)^{\mathrm{T}}\boldsymbol{b} \tag{5}$$

and denote the solution by $\boldsymbol{y}_1^{(i)}$. Adding the elements of $\boldsymbol{y}_1^{(i)}$ to the corresponding elements of $\boldsymbol{x}^{(0)}$ we obtain a vector, labeled $\boldsymbol{x}_1^{(i)}$, that contains p new values of the circuit components. The remaining elements of this vector are the same as in $\boldsymbol{x}^{(0)}$, i.e. they are equal to the nominal values. The obtained $\boldsymbol{x}_1^{(i)}$ is an approximate, specific solution of the test equation. To improve the solution the following iteration process, exploiting the idea of the Newton-Raphson method, is proposed. For $\boldsymbol{x}_1^{(i)}$ we find $\boldsymbol{f}\left(\boldsymbol{x}_1^{(i)}\right)$ and $\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}}\left(\boldsymbol{x}_1^{(i)}\right)$, similarly as we did with $\boldsymbol{x}^{(0)}$ and form up-dated matrix $\boldsymbol{A}_1^{(i)}$ and vector $\boldsymbol{b}_1^{(i)} = \boldsymbol{u} - \boldsymbol{f}\left(\boldsymbol{x}_1^{(i)}\right)$. Then we solve the equation (4), with $\boldsymbol{A}^{(i)}$ replaced by $\boldsymbol{A}_1^{(i)}$ and $\boldsymbol{b}$ replaced by $\boldsymbol{b}_1^{(i)}$, using the equation (5) adapted to this case. The solution is labelled $\boldsymbol{y}_2^{(i)}$, whereas the corresponding parameter vector, $\boldsymbol{x}_2^{(i)}$. The described process is continued, leading to the sequence $\boldsymbol{y}_1^{(i)}, \boldsymbol{y}_2^{(i)}, \ldots, \boldsymbol{y}_l^{(i)}$, where it is assumed that l-th element (vector) $\boldsymbol{y}_l^{(i)}$ of this sequence satisfies a convergence criterion. Next we compute $\tilde{\boldsymbol{y}}^{(i)} = \sum_{j=1}^{l} \boldsymbol{y}_j^{(i)}$. Thus, any element of $\tilde{\boldsymbol{y}}^{(i)}$ is a total perturbation of the corresponding circuit component value.

Since the number of the submatrices $\boldsymbol{A}^{(i)}$, having the rank p, is M we repeat the above described procedure for all of them. As a result a set $\left\{\tilde{\boldsymbol{y}}^{(i)}\right\}$ consisting of $\hat{M}$ elements is created, where $\hat{M}$ can be equal to M or smaller than M, if some of the sequences $\left\{\boldsymbol{y}_1^{(i)}, \boldsymbol{y}_2^{(i)}, \ldots\right\}$ are not convergent in a preset number of iterations $L_{\max}$. The elements of different vectors $\tilde{\boldsymbol{y}}^{(i)}$ of the set $\left\{\tilde{\boldsymbol{y}}^{(i)}\right\}$ correspond to different combinations of p circuit components. Thus, the set $\left\{\tilde{\boldsymbol{y}}^{(i)}\right\}$ consists of $\hat{M}$ elements (vectors) $\tilde{\boldsymbol{y}}^{(i)} = \left[\tilde{y}_{i_1} \ldots \tilde{y}_{i_p}\right]^{\mathrm{T}}$, where $i_1, \ldots i_p$ belong to the set $\{1, 2, \ldots, n\}$.

Procedure for selection of the faulty components

The procedure described underneath enables us to locate up to $p-1$ faulty circuit components and evaluate their approximate values. Let us pick up arbitrarily $p-1$ numbers from the set $\{1, 2, \ldots, n\}$ and select all the p-dimension vectors of the set $\left\{\tilde{\boldsymbol{y}}^{(i)}\right\}$ containing the elements specified by the chosen $p-1$ numbers. Each of the selected p-dimension vectors $\tilde{\boldsymbol{y}}^{(i)}$ contains $p-1$ elements corresponding to the same $p-1$ circuit components and one additional element. Let the number of these vectors be η. We choose all the vectors containing this additional element equal to zero (close to zero) and check if their number $\hat{\eta} \geq c\eta$, where c has been set 0.5 and $\eta \geq 4$ on the basis of numerical experiments. If it holds and values of each of the $p-1$ elements in all the $\hat{\eta}$ vectors are sufficiently close, the $p-1$ circuit components are classified as possibly faulty. Their values are obtained by adding each of the $p-1$ elements to the corresponding element of $\boldsymbol{x}^{(0)}$. The procedure for locating the faulty circuit components and evaluating their values is carried out for all combinations of $p-1$ elements out of n elements, i.e. C_n^{p-1} combinations.

The developed method enables us to find a set of faulty circuit components if their number does not exceed $p-1$. Sometimes the method gives additionally one or more virtual sets that also satisfy the diagnostic test, being the result of existence of some ambiguity groups. Thus, generally several vectors $\boldsymbol{x}$ of the circuit components can be found.

3. Correction of the faults considering the component tolerances

The preliminary method developed in Section 2 enables us to locate faulty components and evaluate their values, under the assumption that the remaining components have nominal values. In reality, however, the components are not nominal but scattered within their tolerance frames. To take into account the influence of the disturbance we correct the obtained results [30].

Let us consider a vector $\boldsymbol{x}$ containing the evaluated values of faulty components and the nominal values of the remaining components, obtained as described in Section 2 and label it $\boldsymbol{x}^*$. Without any loss of generality we can assume that the first $p-1$ elements of the vector $\boldsymbol{x}^*$ correspond to the circuit components that have been diagnosed as faulty. Consequently, the vector $\boldsymbol{x}^*$ can be decomposed into two subvectors: $\boldsymbol{r}^* = \left[r_1^* \cdots r_{p-1}^*\right]^{\mathrm{T}}$ corresponding to the faulty components and $\boldsymbol{z}^* = \left[z_p^* \cdots z_n^*\right]^{\mathrm{T}}$ corresponding to the nominal components,

$$\boldsymbol{x}^* = \begin{bmatrix} \boldsymbol{r}^* \\ \boldsymbol{z}^* \end{bmatrix}. \tag{6}$$

In this section we assume that the non-faulty components are allowed to be arbitrarily dissipated within their tolerance frames and find deviations of the values of the faulty components from $r_1^*, \cdots, r_{p-1}^*$, taking into account the results of the test. Thus, we obtain some ranges of the values of the faulty components near the values $r_1^*, \cdots, r_{p-1}^*$, due to scattering of the values of the non-faulty components within their tolerance frames.

Let the non-faulty components have their values z_j belonging to the tolerance ranges

$$z_j = z_j^* + \Delta z_j, \;\; j = p, \ldots, n, \tag{7}$$

where

$$-\varepsilon_j \le \Delta z_j \le \varepsilon_j. \tag{8}$$

Thus, instead of the vector $\boldsymbol{z}^*$ we consider the vector $\boldsymbol{z} = \left[z_p \cdots z_n\right]^{\mathrm{T}}$. The perturbations of the non-faulty components, within their tolerance frames, cause some perturbations of the obtained values of the faulty components, leading to the vector

$$\boldsymbol{r} = \boldsymbol{r}^* + \Delta\boldsymbol{r}, \tag{9}$$

where: $\Delta\boldsymbol{r} = \left[\Delta r_1 \cdots \Delta r_{p-1}\right]^{\mathrm{T}}$,

$$r_j = r_j^* + \Delta r_j, \;\; j = 1, \ldots, p-1. \tag{10}$$

Thus, we need the lower and upper bounds on Δr_j, $(j = 1, \ldots, p-1)$, labelled Δr_j^- and Δr_j^+ respectively, so that

$$\Delta r_j^- \le \Delta r_j \le \Delta r_j^+. \tag{11}$$

For this purpose we use the linear programming approach.

Let us define some range $[-\delta_j, \delta_j]$ in which Δr_j is sought

$$-\delta_j \le \Delta r_j \le \delta_j,\ j = 1,\ldots, p-1 \tag{12}$$

and introduce new variables $\Delta\tilde{r}_j$

$$\Delta\tilde{r}_j = \Delta r_j + \delta_j, \tag{13}$$

which, according to (12) satisfy the inequalities

$$0 \le \Delta\tilde{r}_j \le 2\delta_j,\ j = 1,\ldots, p-1. \tag{14}$$

Similarly we introduce new variables $\Delta\tilde{z}_j$

$$\Delta\tilde{z}_j = \Delta z_j + \varepsilon_j, \tag{15}$$

which, according to (8), satisfy the inequalities

$$0 \le \Delta\tilde{z}_j \le 2\varepsilon_j,\ j = p,\ldots, n. \tag{16}$$

On the basis of the test equation (1) we write

$$\hat{\boldsymbol{f}}(\boldsymbol{x}^*) + \frac{\partial\hat{\boldsymbol{f}}}{\partial\boldsymbol{r}}(\boldsymbol{x}^*)(\boldsymbol{r}-\boldsymbol{r}^*) + \frac{\partial\hat{\boldsymbol{f}}}{\partial\boldsymbol{z}}(\boldsymbol{x}^*)(\boldsymbol{z}-\boldsymbol{z}^*) = \hat{\boldsymbol{u}}, \tag{17}$$

where: $\boldsymbol{x}^* = \left[(\boldsymbol{r}^*)^{\mathrm{T}}\ (\boldsymbol{z}^*)^{\mathrm{T}}\right]^{\mathrm{T}}$, $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{u}}$ are $\hat{n}$-vectors selected from $\boldsymbol{f}$ and $\boldsymbol{u}$, respectively, where $\hat{n} < n$. To choose $\hat{n}$ and select $\hat{n}$ appropriate equations from the system of n test equations we use the following procedure. First we determine the rank of the matrix $\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}(\boldsymbol{x}^*)$ and label it $\bar{n}$. If $\bar{n} = n$ then $\hat{n} = n-1$, otherwise $\hat{n} = \bar{n}$. Then, we consider all combinations of $\hat{n}$ equations from among n ones and select this combination which fulfils two requirements: $\operatorname{rank}\left(\frac{\partial\hat{\boldsymbol{f}}}{\partial\boldsymbol{x}}(\boldsymbol{x}^*)\right) = \hat{n}$ and $\left\|\hat{\boldsymbol{u}} - \hat{\boldsymbol{f}}(\boldsymbol{x}^*)\right\|_2$ is minimal.

The equation (17) can be written in the form

$$\sum_{j=1}^{p-1} s_{ij}\Delta r_j + \sum_{j=p}^{n} s_{ij}\Delta z_j = d_i,\ i = 1,\ldots,\hat{n}, \tag{18}$$

where: $d_i = \hat{u}_i - \hat{f}_i(\boldsymbol{x}^*)$, $i = 1,\ldots,\hat{n}$, and

$$s_{ij} = \begin{cases} \dfrac{\partial\hat{f}_i}{\partial r_j}(\boldsymbol{x}^*) & j = 1,\ldots, p-1, \\ \dfrac{\partial\hat{f}_i}{\partial z_j}(\boldsymbol{x}^*) & j = p,\ldots, n. \end{cases} \tag{19}$$

Substituting $\Delta r_j = \Delta\tilde{r}_j - \delta_j$ (see(13)) and $\Delta z_j = \Delta\tilde{z}_j - \varepsilon_j$ (see (15)) into (18) yields

$$\sum_{j=1}^{p-1} s_{ij}\Delta\tilde{r}_j + \sum_{j=p}^{n} s_{ij}\Delta\tilde{z}_j = \tilde{d}_i,\ i = 1,\ldots,\hat{n}, \tag{20}$$

where:

$$\tilde{d}_i = d_i + \sum_{j=1}^{p-1} s_{ij}\delta_j + \sum_{j=p}^{n} s_{ij}\varepsilon_j \,. \tag{21}$$

We select all the equations of the set (20) having $\tilde{d}_i < 0$ and multiply them by (-1). As a result we obtain a system of equations

$$\sum_{j=1}^{p-1} \hat{s}_{ij}\Delta\tilde{r}_j + \sum_{j=p}^{n} \hat{s}_{ij}\Delta\tilde{z}_j = \hat{d}_i \,,\ i = 1,\ldots,\hat{n}\,, \tag{22}$$

where $\hat{d}_i \geq 0$ $(i = 1,\ldots,\hat{n})$. Let us introduce the slack variables $\tilde{w}_j \geq 0$ $(j = 1,\ldots,n)$ so that

$$\Delta\tilde{r}_j + \tilde{w}_j = 2\delta_j \,,\ j = 1,\ldots, p-1\,, \tag{23}$$

$$\Delta\tilde{z}_j + \tilde{w}_j = 2\varepsilon_j \,,\ j = p,\ldots,n\,. \tag{24}$$

For each $j \in \{1,\ldots, p-1\}$ we formulate the linear programming problem:

$$\text{maximize } \Delta\tilde{r}_j$$

$$\text{subject to} \quad \begin{aligned} &\sum_{j=1}^{p-1} \hat{s}_{ij}\Delta\tilde{r}_j + \sum_{j=p}^{n} \hat{s}_{ij}\Delta\tilde{z}_j = \hat{d}_i \,,\ i = 1,\ldots,\hat{n}\,,\\ &\Delta\tilde{r}_j + \tilde{w}_j = 2\delta_j \,,\ j = 1,\ldots, p-1\,,\\ &\Delta\tilde{z}_j + \tilde{w}_j = 2\varepsilon_j \,,\ j = p,\ldots,n\,,\\ &\Delta\tilde{r}_j \geq 0,\ \Delta\tilde{z}_j \geq 0,\ \tilde{w}_j \geq 0 \end{aligned} \tag{25}$$

and solve it using the simplex method [32-33]. As a result we find: $(\Delta\tilde{r}_1)_{max},\ldots,(\Delta\tilde{r}_{p-1})_{max}$. Similarly we find $(\Delta\tilde{r}_1)_{min},\ldots,(\Delta\tilde{r}_{p-1})_{min}$ minimizing $\Delta\tilde{r}_j$ for $j = 1,2,\ldots, p-1$. Using (13), yields

$$\Delta r_j^{+} = (\Delta r_j)_{max} = (\Delta\tilde{r}_j)_{max} - \delta_j \,,\ j = 1,\ldots, p-1\,, \tag{26}$$

$$\Delta r_j^{-} = (\Delta r_j)_{min} = (\Delta\tilde{r}_j)_{min} - \delta_j \,,\ j = 1,\ldots, p-1\,. \tag{27}$$

Thus, we find the region including the values of the faulty components: $(\boldsymbol{r}^* + \Delta\boldsymbol{r}^-,\ \boldsymbol{r}^* + \Delta\boldsymbol{r}^+)$, where $\Delta\boldsymbol{r}^- = [\Delta r_1^- \cdots \Delta r_{p-1}^-]^{\mathrm{T}}$, $\Delta\boldsymbol{r}^+ = [\Delta r_1^+ \cdots \Delta r_{p-1}^+]^{\mathrm{T}}$, valid for actual values of the non-faulty components belonging to their tolerance ranges.

4. Numerical examples

The proposed method was implemented in MATLAB and tested using several transistor circuits. Two of them are presented below. The calculations were executed on a PC Pentium Core 2 Duo E 6400.

Example 1

Consider the circuit shown in Fig. 1 [30], where nominal values of the circuit components are indicated, and their tolerance is 5%. All seven resistors of the circuit are considered as possibly faulty. We assume that the nodes A and B are accessible for measurement. The transistors are characterized by the Ebers-Moll model with the following parameters: $\alpha_F = 0.9975$, $\alpha_R = 0.8$, $I_{ES} = 10.22$ fA, $I_{CS} = 12.75$ fA, $V_T = 25.86$ mV. To perform a diagnostic test we measure voltages at the test nodes A and B at the three combinations of the voltage sources: $v_{in}^{(1)} = 15$ V, $v_{in}^{(2)} = 15$ V; $v_{in}^{(1)} = 5$ V, $v_{in}^{(2)} = 15$ V; $v_{in}^{(1)} = 15$ V, $v_{in}^{(2)} = 0$ V and at the test node B for $v_{in}^{(1)} = 6$ V, $v_{in}^{(2)} = 6$ V, assuming an accuracy of 10 μV. Underneath, four cases are described assuming $\varepsilon_j = 0.05 z_j^*$ and $\delta_j = 0.20 r_j^*$ in the correction procedure.

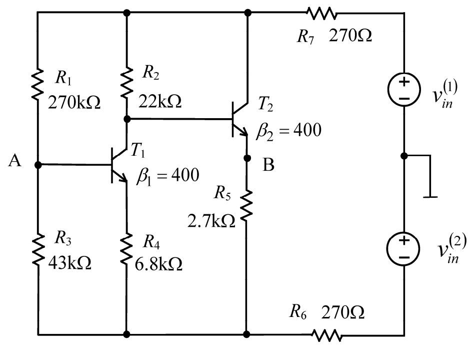

Fig. 1. A transistor circuit for Example 1.

Case 1

Components $R_1 = 175.5$ kΩ, $R_2 = 14.30$ kΩ, $R_6 = 175.5$ Ω are faulty $(-35\%, -35\%, -35\%)$ and the others are within their tolerance ranges: $R_3 = 44.2$ kΩ, $R_4 = 6.95$ kΩ, $R_5 = 2.78$ kΩ, $R_7 = 277.0$ Ω. The preliminary method leads to the correct set of the faulty components $\{R_1, R_2, R_6\}$. The correction procedure, described in Section 3, gives the following results: $R_1 \in (162.2 - 177.4)$ kΩ, $R_2 \in (13.26 - 14.71)$ kΩ, $R_6 \in (162.8 - 180.2)$ Ω, including the actual values of the components.

Case 2

Components $R_3 = 21.5$ kΩ, $R_4 = 3.4$ kΩ, $R_6 = 135$ Ω are faulty $(-50\%, -50\%, -50\%)$ and the others are within their tolerance ranges: $R_1 = 265.0$ kΩ, $R_2 = 21.0$ kΩ, $R_5 = 2.78$ kΩ, $R_7 = 277.0$ Ω. The preliminary method leads to the correct set of the faulty components $\{R_3, R_4, R_6\}$, whereas the correction procedure gives the following results: $R_3 \in (20.80 - 22.18)$ kΩ, $R_4 \in (3.40 - 3.42)$ kΩ, $R_6 \in (133.67 - 137.82)$ Ω, including the actual values of the components.

Case 3

Components $R_3 = 58.05$ kΩ, $R_4 = 9.18$ kΩ, $R_7 = 364.5$ Ω are faulty $(35\%, 35\%, 35\%)$ and the others are within their tolerance ranges: $R_1 = 265.0$ kΩ, $R_2 = 21.0$ kΩ, $R_5 = 2.78$ kΩ, $R_6 = 278.0$ Ω. The preliminary method leads to three sets of components: $\{R_1, R_4, R_7\}$, $\{R_2, R_3, R_7\}$, and $\{R_3, R_4, R_7\}$. The last is the real one, whereas the others are virtual. The correction procedure eliminates the first set and gives the following results: $R_2 \in (14.66-16.24)\,\text{k}\Omega$, $R_3 \in (57.82-62.56)\,\text{k}\Omega$, $R_7 \in (363.8-402.7)\,\Omega$ and $R_3 \in (56.12-59.10)\,\text{k}\Omega$, $R_4 \in (9.14-10.08)\,\text{k}\Omega$, $R_7 \in (356.7-373.5)\,\Omega$. Thus, the method provides the ranges of the actual values of the faulty components and another ranges of values of the virtual set of components.

Case 4

Components $R_1 = 540\,\text{k}\Omega$, $R_2 = 44\,\text{k}\Omega$, $R_5 = 5.4\,\text{k}\Omega$ are faulty $(100\%, 100\%, 100\%)$ and the others are within their tolerance ranges: $R_3 = 44.2\,\text{k}\Omega$, $R_4 = 6.95\,\text{k}\Omega$, $R_6 = 278.0\,\Omega$, $R_7 = 277.0\,\Omega$. The preliminary method leads to the correct set of the faulty components $\{R_1, R_2, R_5\}$. The correction procedure gives the following results: $R_1 \in (498.68-551.74)\,\text{k}\Omega$, $R_2 \in (40.81-45.22)\,\text{k}\Omega$, $R_5 \in (5.00-5.50)\,\text{k}\Omega$, including the actual values of the components.

Some statistical results

For statistical analysis 112 sets of faulty components $(\pm 35\%)$ were considered, including all combinations of double (42) and triple (70) faults. The method found the faulty set in 83.0%, in 59.8% uniquely. In each case the provided ranges of values of the faulty components effectively framed the actual values. In 12.5% the preliminary method failed. In 4.5% of the cases, values evaluated by the preliminary method were far away from the actual values and they were discarded during the first phase of the simplex method. The CPU time in each case was less than 5s.

Example 2

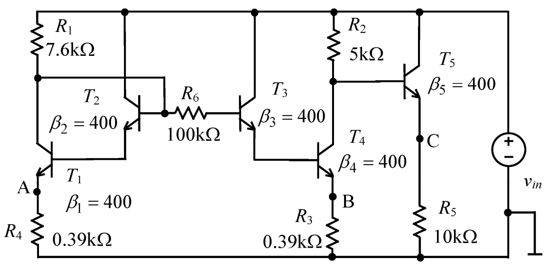

Fig. 2. A transistor circuit for Example 2.

Let us consider the transistor circuit shown in Fig. 2, where nominal values of the circuit components are indicated. The tolerance of the components is 5%. All six resistors of the circuit are considered as possibly faulty. We assume that the nodes A, B, and C are accessible for measurement. The transistors are characterized by the Ebers-Moll model with the following parameters: $\alpha_F = 0.9975$, $\alpha_R = 0.8$, $I_{ES} = 10.22\,\text{fA}$, $I_{CS} = 12.75\,\text{fA}$, $V_T = 25.86\,\text{mV}$. To perform a diagnostic test we measure voltages at the test nodes for $v_{in}^{(1)} = 12\,\text{V}$ and $v_{in}^{(2)} = 5\,\text{V}$, assuming the accuracy to be $1\,\mu\text{V}$. We wish to diagnose double

$(p=3)$ and triple $(p=4)$ faults. Underneath, three cases are described assuming in the correction procedure $\varepsilon_j = 0.05 z_j^*$ and $\delta_j = 0.20 r_j^*$.

Case 1

Components $R_1 = 4.94\ \mathrm{k\Omega}$, $R_4 = 253.5\ \Omega$ are faulty $(-35\%, -35\%)$ and the others are within their tolerance ranges: $R_2 = 4.87\ \mathrm{k\Omega}$, $R_3 = 385.0\ \Omega$, $R_5 = 9.89\ \mathrm{k\Omega}$, $R_6 = 102\ \mathrm{k\Omega}$. The preliminary method, described in Section 2, leads to two sets of faulty components $\{R_1, R_4\}$ and $\{R_2, R_3\}$, where the first set is true and the other is virtual. During the correction procedure, described in Section 3, the second set is eliminated. The correction procedure gives the following ranges of values of the faulty components: $R_1 \in (4.940 - 4.941)\ \mathrm{k\Omega}$, $R_4 \in (253.5 - 253.6)\ \Omega$, that very well frame the actual values.

Case 2

Components $R_1 = 12.0\ \mathrm{k\Omega}$, $R_2 = 9.00\ \mathrm{k\Omega}$, and $R_5 = 13.5\ \mathrm{k\Omega}$ are faulty $(58\%, 80\%, 35\%)$ and the others are within their tolerance ranges: $R_3 = 385.0\ \Omega$, $R_4 = 392.0\ \Omega$, $R_6 = 102\ \mathrm{k\Omega}$. The preliminary method gives the correct set of faulty components $\{R_1, R_2, R_5\}$. The correction procedure leads to the following ranges of values of the faulty components: $R_1 \in (11.999 - 12.002)\ \mathrm{k\Omega}$, $R_2 \in (8.984 - 9.018)\ \mathrm{k\Omega}$, $R_5 \in (10.790 - 16.184)\ \mathrm{k\Omega}$, including the actual values of the components.

Case 3

Components $R_4 = 253.5\ \Omega$, $R_5 = 6.5\ \mathrm{k\Omega}$ are faulty $(-35\%, -35\%)$ and the others are within their tolerance ranges: $R_1 = 7.4\ \mathrm{k\Omega}$, $R_2 = 4.87\ \mathrm{k\Omega}$, $R_3 = 385.0\ \Omega$, $R_6 = 102\ \mathrm{k\Omega}$. The preliminary method, described in Section 2, leads to four sets of faulty components $\{R_1, R_4\}$, $\{R_2, R_4\}$, $\{R_3, R_4\}$, and $\{R_4, R_5\}$, where the last one is true and the others are virtual. During the correction procedure, the second set is eliminated leading to the following results: $R_1 \in (7.3970 - 7.3973)\ \mathrm{k\Omega}$, $R_4 \in (253.39 - 253.40)\ \Omega$; $R_3 \in (384.45 - 384.47)\ \Omega$, $R_4 \in (253.30 - 253.31)\ \Omega$; $R_4 \in (253.35 - 253.44)\ \Omega$, $R_5 \in (4.740 - 7.110)\ \mathrm{k\Omega}$, where the last set is true and the others are virtual.

Some statistical results

In the discussed circuit 40 sets of faulty circuit components were considered, including 25 double and 15 triple faults. The method found the faulty set in 87.5%, in 72.5% uniquely. In each case the provided ranges of values of the faulty components effectively framed the actual values. In 12.5% the preliminary method failed. The CPU time in each case was between 2 and 10 seconds.

5. Conclusions

The two-stage algorithm developed in this paper enables us to perform multiple soft fault diagnosis of DC nonlinear circuits, even if the deviations of the faulty components from their nominal values are large. The diagnosis includes location of faulty components and evaluation of their values, considering the component tolerances. The results are given in the form of some ranges where the values of the faulty components are contained. The numerical experiments carried out show that the evaluated ranges effectively frame the actual values of the faulty components. The method can be extended to dynamic circuits. The proposed approach is especially useful at the pre-production stage, where corrections of the technological process are possible and the time consumed by the diagnostic procedure is not crucial.

Appendix

Creating the submatrices $\boldsymbol{A}^{(i)}$ having the rank p

To determine the rank of the $n \times n$ matrix $\boldsymbol{A}$ the singular value decomposition (SVD) [31] is applied

$$\boldsymbol{A} = \boldsymbol{U\Sigma V}^{\mathrm{T}}, \tag{A.1}$$

where: $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal $n \times n$ matrices, $\boldsymbol{\Sigma}$ is an $n \times n$ matrix of the form

$$\boldsymbol{\Sigma} = \left[\begin{array}{ccc|c} \sigma_1 & & & \\ & \ddots & & \mathbf{0} \\ & & \sigma_r & \\ \hline & \mathbf{0} & & \mathbf{0} \end{array}\right], \tag{A.2}$$

where: $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$ are called singular values of $\boldsymbol{A}$ and r is the rank of $\boldsymbol{A}$ $(r = \operatorname{rank}(\boldsymbol{A}))$. Note that

$$\boldsymbol{\Sigma V}^{\mathrm{T}} = \left[\begin{array}{c} \boldsymbol{L} \\ \hline \mathbf{0} \end{array}\right], \tag{A.3}$$

where: $\boldsymbol{L}$ is an $r \times n$ matrix and $\mathbf{0}$ is $(n-r)\times n$ zero matrix. Taking into account the equations (A.1) and (A.3) we obtain

$$\boldsymbol{A} = \boldsymbol{KL}, \tag{A.4}$$

where: $\boldsymbol{K}$ is an $n \times r$ matrix composed of the first r columns of $\boldsymbol{U}$. Since $\operatorname{rank}(\boldsymbol{A}) = r$, the matrix $\boldsymbol{K}$ has also the rank r. We consider all $r \times p$ submatrices of the $r \times n$ matrix $\boldsymbol{L}$ and select these of them which have the rank equal to p using SVD. Let us denote the submatrices by $\boldsymbol{L}^{(i)}$, $i = 1,\ldots,M$. For an arbitrary $r \times p$ submatrix $\boldsymbol{L}^{(i)}$ the corresponding $n \times p$ submatrix $\boldsymbol{A}^{(i)}$ of $\boldsymbol{A}$, $\boldsymbol{A}^{(i)} = \boldsymbol{KL}^{(i)}$ has the rank equal to p (see Lemma 1).

Thus, to find all the $n \times p$ submatrices $\boldsymbol{A}^{(i)}$ of $\boldsymbol{A}$ having the rank p we operate with smaller submatrices $\boldsymbol{L}^{(i)}$. For each $\boldsymbol{L}^{(i)}$ we check which columns of $\boldsymbol{L}$ have been selected to form $\boldsymbol{L}^{(i)}$ and use the same columns of $\boldsymbol{A}$ to create $\boldsymbol{A}^{(i)}$. In this way the procedure for creating the $n \times p$ submatrices $\boldsymbol{A}^{(i)}$ having the rank p is simplified.

Lemma 1

Let us consider an $n \times r$ matrix $\boldsymbol{K}$ $(n \geq r)$ and an $r \times p$ matrix $\boldsymbol{S}$, where $\operatorname{rank}(\boldsymbol{K}) = r$, $\operatorname{rank}(\boldsymbol{S}) = p$. Then the matrix $\boldsymbol{KS}$ has the rank equal to p.

Proof

Let $\boldsymbol{S} = [\boldsymbol{s}_1 \ldots \boldsymbol{s}_p]$, $\boldsymbol{K} = [\boldsymbol{k}_1 \ldots \boldsymbol{k}_r]$, where $\boldsymbol{s}_i$ is *i*-th column of the matrix $\boldsymbol{S}$ and $\boldsymbol{k}_i$ is *i*-th column of the matrix $\boldsymbol{K}$. Then $\boldsymbol{KS} = [\boldsymbol{Ks}_1 \ldots \boldsymbol{Ks}_p]$ holds. Suppose that there exist such numbers $c_1, \ldots, c_p$, not all equal to zero, that $c_1(\boldsymbol{Ks}_1) + \ldots + c_p(\boldsymbol{Ks}_p) = \boldsymbol{K}(c_1\boldsymbol{s}_1 + \ldots + c_p\boldsymbol{s}_p) = \boldsymbol{0}$. Since $\boldsymbol{S}$ has rank p, its columns are linearly independent, hence, $c_1\boldsymbol{s}_1 + \ldots + c_p\boldsymbol{s}_p \neq \boldsymbol{0}$. Thus, $\boldsymbol{w} = [w_1 \ldots w_r]^{\mathrm{T}} = c_1\boldsymbol{s}_1 + \ldots + c_p\boldsymbol{s}_p \neq \boldsymbol{0}$ and $w_1\boldsymbol{k}_1 + \ldots + w_r\boldsymbol{k}_r = \boldsymbol{0}$, where not all numbers $w_1, \ldots, w_r$ are equal to zero. This is a contradiction because $\boldsymbol{K}$ has, by the assumption, rank r.

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